

Reading the Torah with Equal Intervals

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What is equal interval reading?

Let us eliminate the spaces between the words and consider the text as a sequence of letters. Now, starting from a certain letter let us skip N-1 letters and read the Nth one, again skip N-1 letters and read the Nth one and so on. This will be called a reading with the interval N. The number N may also be negative in which case the reading is backwards. Of course, besides the interval N one has to know where to start counting and how many intervals to go.

Let us make things more clear by showing a few examples. If one starts with the first letter **ת** in the Genesis (i.e. the **ת** of **בראשית**) and skips 49 letters one arrives at the letter **ו** in **תהום**, again skips 49 letters and arrives at **ר** in **ויראה**, again skips 49 letters and arrives at **ה** in **אלהים** . Thus one finds that the word **תורה** (Torah) is spelled out with interval 50 right in the beginning of Genesis. See Figure 1.

1/01.01 תברא אלהימ אתהשמי ואתהארצו והארצה יתהתהו ובהו וחשכעל 1/01.02
1/01.03 פניתהו ורוחאלהימ מרחפתעלפניהמי ויאמראלהימ יהיאוורויהי 1/01.04
1/01.05 ויראו ויראאלהימ אתהאורכיסובו יבדלאלהימבנהאורובינהחשכו 1/01.06
1/01.06 יאמאחדיומאוריומלאוריומולחשכקראלילהויערבויהיבקריומאחדיו

Figure 1 shows the first skip 50 ELS of the key word תורה, Torah on a cylinder size of 50.

The number 50 has several important meanings in Judaism. The 50th is the Jubilee year, there are 49 days of Omer which are counted from the second day of Passover until the Shavuot which is the 50th day, and there are also 50 gates of wisdom in Torah.

The above example is a part of a bigger pattern found by Rabbi Michael Weissmandel about 40 years ago. Namely, in the second book Exodus, the word **תורה** is again spelled out with the interval 50 beginning with the very first letter **ת** in the book (i.e. the **ת** of **שמות**). In the fourth book Numbers, the word **תורה** is spelled out with the interval 50, i.e. backwards with the letter H starting in the first verse of the book. Finally, in the last book Deuteronomy, the word **תורה** is spelled again backwards however with interval 49 instead of 50 and the letter **ה** starts in the 5th verse instead of the first.

Why this deviation and why is there no **תורה** in the third book Leviticus? Gaon from Vilna wrote in Aderet Eliahu that Deuteronomy actually starts from the 5th verse, while the first four verses correspond to the first four books. Indeed, the fifth verse reads: "On the other side of the Jordan, in the land of Moab, Moses undertook to expound the Torah. He said . " It is claimed that Moses was given 49 gates of wisdom instead of fifty. Since the subsequent explanation of Torah is given from the mouth of Moses, the word **תורה** is spelled out with the interval 49.

We see that the system is symmetric in the first two books **תורה** is spelled out forwards and in the last two books backwards. Hence, in the central book Leviticus you don't find the word **תורה**. Instead, the four letter name of Gd who gave the Torah is spelled out with interval 8 starting with the very first letter ' of Leviticus. (the number $8=7+1$ is closely related to $50=7 \times 7+1$ but this is a separate story on which we will not elaborate here).

At this point a skeptical reader would exclaim that the whole system is nothing but a coincidence and the above explanation with 50 and 49 gates of wisdom was "cooked up" to tie several unrelated appearances of the word **תורה** into a system. "I'm sure", this skeptic would continue, "you would be able to find such words and systems in any book". Since the author of this review was, until recently, such a skeptic the question of coincidence versus intentional design will be addressed most forcefully in this article. Meanwhile let us mention that on the statistical basis the word **תורה** is expected to appear with any given interval N in Genesis about 2 or 3 times. This estimate is based on the total number of letters in Genesis (78064) and on the number of the letters **ת** (4152), **ו** (8448), **ר** (4793) and **ה** (6283) in the book. Indeed, **תורה** appears 3 times in Genesis with the interval 50 which is what one would expect from any book of such length and of similar concentration of letters **ה,ו,ר,ת**. There is however no reason why one of these three appearances should start with the very first **ת** of the book and why this should happen both in Genesis and Exodus. As a matter of fact the probability of such a coincidence is about 1 in 3 million!

The above is one of hundreds of patterns found by Rabbi Weissmandel in Torah in the time of W.W.II. After his death in 1957 his students published in the the book "Torat Chemed" where just a handful of his findings were exhibited. The rest of the findings were lost. Of course at that time there were no computers. Instead Rabbi Weissmandel was guided by a deep knowledge of Torah as for what to seek and where to seek.

As for the length of the intervals most of his examples refer to the numbers 50 or 26, the last being the Gematria of the four letter name of God (**יהוה** = $10+5+6+5 = 26$).

Later on, a few followers of Rabbi Weissmandel continued the search, which was still done by hand. We should mention Rabbi Shmuel Yaniv, Abraham Oren and their students. But the real breakthrough occurred in 1982 when the computer was put to work. Here most of the credit should be given to Dr. Eli Rips from Institute of Mathematics, Hebrew University who was joined by Dr. Moshe Katz from the Technion, Haifa and later on by Doron Witztum from Jerusalem.

Let us make it clear the computer does not have an intelligence to find meaningful patterns. Instead it is used as a fast and accurate counting machine. The text being investigated is typed into the computer and is stored there as a file of integers. A set of instructions would then tell the computer to look for a certain word in the text with equal intervals in a given range.

For example, find all appearances of the word **ישראל**, Israel (which in integer form spells 10 21 20 1 12) in the first 10000 letters of Genesis, with equal intervals ranging from 100 to 100. The computer then shows that the word is spelled out only twice, the intervals being 7 and 50 and is

located in the four verses 1:31 through 2:3 See Figure 2.

1/01:30 ולכלע ופהשמ ימ ולכלר ומשע להארצאשרבו נפשח יהאתכל ירקעשב 1/01:30
 1/01:31 לאכלה ויה יכנו יראאלה ימאתכל אשרעשה והנהט ובמדזו יה יערבו ויה 1/01:31
 1/02:02 יבקר יומהשש יוכל והשמ ימ והארצ וכלצב אמ ויכלא לה ימבי ומהשב י 1/02:02
 1/02:03 ימלאכת ואשרעשה וישבתב יומהשב יע ימכלמלאכת ואשרעשה ויברכא 1/02:03
 1/02:03 לה ימאת יומהשב יע יקדשאת וכ יב ושבתמכלמלאכת ואשרבראאלה ימ 1/02:03

Figure 2 show the skip -50 and skip 7 ELS of the key word ישראל Israel on a cylinder size of 50.

We are stunned by the fact that these verses constitute our Kiddush recited every Sabbath evening over a cup of wine. Indeed 7 and 50 are the only numbers related to the Sabbath. The number 7 stands for the seventh day of creation and also for the seventh year the year of Shmita when the land rests. Then, after 7 Shmita cycles the land should rest also on the Jubilee year which is the 50th year. Again, a coincidence? A simple calculation shows that the probability of the word **ישראל** to appear once with a given interval in the above verses is about 1 in 1200. The chance of two appearances with intervals of 7 and 50 either backwards or forwards is about 1 in 400,000.

Another interesting example is shown in fig.3. The text which is Genesis 38 tells the story of Yehuda and Tamar. As the result of their affair, Tamar gave birth to Perez and Zerach. From the book of Ruth we learn that Perez started a lineage which led to Boaz. Boaz married Ruth and had a son Oved, which had a son Yishai, which was the father of King David. So it was a natural question to ask whether King David with his lineage is hidden in this chapter. Indeed, you find the names **בעז, רות, עבד, ישי** and **זוד** spelled out with the same skip interval -49, Moreover they all appear in the chronological order! We already mentioned the importance of 49 being the 7th Shmita followed by the Jubilee. However 49 is also the last day of the counting of Omer which starts on the second day of the Passover and ends a day before Shavuot. Every day in this counting has a name and the 49th is called **מלכות במלכות**, kingdom of the kingdom. Is there a name which would fit David, the king of kings, better? Let us also mention that David was born and died on the very day of Shavuot and the book of Ruth is traditionally studied on this holiday.

But maybe this system is another coincidence? It is easy to estimate the probability of such an event. As we count the total number of letters in Gen. 38 and the relative proportion of each of the letters of the alphabet, we come to the conclusion that the probability of the word **בעז**, Boaz, to appear in our chapter with a given interval is 0.02. (That is assuming that on the level of equal intervals the text is random). Similarly, for the other four names the probabilities are 0.63, 0.065, 0.76 and 0.25. The odds for all 5 names to show up with a given interval are about 1 in 6,500. If we also request that the names line up in chronological order, the chances are reduced to 1 in 800,000. Now, if one would claim that the interval 49 is as important as 49 and the same for 50 and 50, these 3 possibilities would increase the chances to 1 in 200,000 still quite an impressive number!

1/38:09 י לא ל ו י ה י ה ז 1/38:09
 1/38:09 ת א ח י ו ו ש ח ת א ר 1/38:09
 1/38:10 א ח י ו ו י ר ע ב ע י 1/38:09
 1/38:11 מ ת ג מ א ת ו ו י א מ 1/38:10
 1/38:11 ש ב י א ל מ נ ה ב י ת 1/38:11
 1/38:11 נ י כ י א מ ר פ נ י מ 1/38:11
 1/38:11 ל כ ת מ ר ו ת ש ב ב י 1/38:11
 1/38:12 מ ו ת מ ת ב ת ש ו נ ע א 1/38:12
 1/38:12 ו ד ה ו י ע ל ע ל ג ז 1/38:12
 1/38:12 ר ע ה ו ה ע ד ל מ י ת 1/38:12
 1/38:13 מ ר ה נ ה ח מ י כ ע ל 1/38:13
 1/38:14 ת ס ר ב ג ד י א ל מ נ 1/38:14
 1/38:14 ע י פ ו ת ת ע ל פ ו ת 1/38:14
 1/38:14 ע ל ד ר כ ת מ נ ת ה כ 1/38:14
 1/38:14 ה ו א ל א נ ת נ ה ל ו 1/38:14
 1/38:15 ו י ח ש ב ה ל ו נ ה 1/38:15
 1/38:16 ל י ה א ל ה ד ר כ ו י 1/38:16
 1/38:16 י כ כ י ל א י ד ע כ י 1/38:16
 1/38:16 ת ת נ ל י כ י ת ב ו א 1/38:16
 1/38:17 ל ח ג ד י ע ז י מ מ נ 1/38:17
 1/38:17 ע ר ב ו נ ע ד ש ל ח כ 1/38:17
 1/38:18 ש ר א ת נ ל כ ו ת א מ 1/38:18
 1/38:18 כ א ש ר ב י ד כ ו י ת 1/38:18
 1/38:19 ר ל ו ו ת ק מ ו ת ל כ 1/38:18
 1/38:19 ו ת ל ב ש ב ג ד י א ל 1/38:19
 1/38:20 ה א ת ג ד י ה ע ז י מ 1/38:20
 1/38:20 ק ח ת ה ע ר ב ו נ מ י 1/38:20
 1/38:21 ש א ל א ת א נ ש י מ ק 1/38:21
 1/38:21 ה ו א ב ע י נ י מ ע ל 1/38:21
 1/38:22 ת ה ב ז ה ק ד ש ה ו י 1/38:21
 1/38:22 ל א מ צ א ת י ה ו ג מ 1/38:22
 1/38:22 א ה י ת ה ב ז ה ק ד ש 1/38:22
 1/38:23 ל ה פ נ נ ה י ה ל ב ו 1/38:23
 1/38:23 ז ה ו א ת ה ל א מ צ א 1/38:23
 1/38:24 י מ ו י ג ד ל י ה ו ד 1/38:24
 1/38:24 ת כ ו ג מ ה נ ה ה ר ה 1/38:24
 1/38:24 ו ד ה ה ו צ י א ו ה ו 1/38:24
 1/38:25 י א ש ל ח ה א ל ח מ י 1/38:25
 1/38:25 ה ל ו א נ כ י ה ר ה ו 1/38:25
 1/38:25 ת מ ת ו ה פ ת י ל י מ 1/38:25
 1/38:26 ה ו ד ה ו י א מ ר צ ד 1/38:26
 1/38:26 נ ת ת י ה ל ש ל ה ב נ 1/38:26
 1/38:27 ה ו י ה י ב ע ת ל ד ת 1/38:26
 1/38:28 נ ה ו י ה ב ל ד ת ה 1/38:27
 1/38:28 ד ת ו ת ק ש ר ע ל י ד 1/38:28

Figure 3 The cylinder size is 49. The table shows absolute skip 49 ELSs for Boaz, Ruth, Oved, Yishai, and David, all in correct chronological order.

Let us turn to the third example in Figure 4. We are in the beginning of the Parasha **ויצא** where it talks about the famous dream of Jacob with the stairway reaching the sky. As Jacob awoke from his sleep he said, "Surely the Lord is present in this place, and I did not know it!" (Genesis 28:16). Where was this place? Rashi (the main commentator of the Torah) writes that this was Mount Moriah where the Temple was built later on. Moshe Katz who was reading the

commentaries decided to check for the word **מקדש**, Temple. Indeed, the word does appear with a very important skip interval 26 starting with the **מ** of the word **מקום** (place) in the above verse. However as you continue to count 26 letter intervals after **ש** of **מקדש** you find another five letter word **התורה** (the Torah) spelled forwards. Thus the two cornerstones of Judaism **התורה** and **מקדש** are spelled as one continuous sequence of 9 letters with the interval 26 (which is, to repeat, the numerical value of Lord's name). The probability of such an event (for a fixed position of the first letter **מ**) is about 1 in 17 billion! In the same story we also find the words **ציון** (Zion) and **מקום** (place) spelled out with the interval 26.

1 / 28 : 13	ר	ה	מ	א	ב	י	כ	ו	א	ל	ה	י	צ	ח	ק	ה	א	ר	צ	1 / 28 : 13	
1 / 28 : 13	ש	כ	ב	ע	ל	י	ה	ל	כ	א	ת	נ	נ	ה	ו	ל	ז	ר	ע	כ	1 / 28 : 13
1 / 28 : 14	ע	כ	כ	ע	פ	ר	ה	א	ר	צ	ו	פ	ר	צ	ת	י	מ	ה	ו	ק	1 / 28 : 14
1 / 28 : 14	נ	ה	ו	נ	ג	ב	ה	ו	נ	ב	ר	כ	ו	ב	כ	ל	מ	ש	פ	1 / 28 : 14	
1 / 28 : 15	ה	ו	ב	ז	ר	ע	כ	ו	ה	נ	ה	א	נ	כ	י	ע	מ	כ	ו	ש	1 / 28 : 15
1 / 28 : 15	כ	ל	א	ש	ר	ת	ל	כ	ו	ה	ש	ב	ת	י	כ	א	ל	ה	א	ד	1 / 28 : 15
1 / 28 : 15	כ	י	ל	א	ע	ז	ב	כ	ע	ד	א	ש	ר	א	מ	ע	ש	י	ת	1 / 28 : 15	
1 / 28 : 16	ד	ב	ר	ת	י	ל	כ	ו	י	י	ק	צ	י	ע	ק	ב	מ	ש	נ	ת	1 / 28 : 16
1 / 28 : 16	א	כ	נ	י	ש	י	ה	ו	ה	ב	מ	ק	ו	מ	ה	ז	ה	ו	א	נ	1 / 28 : 16

Figure 4. The cylinder size is 26. The table shows ELSs of the key words **מקדש, Temple and **התורה**, the Torah.**

The next example in Figure 5 (found by Moshe Katz) is related to Joseph's second dream (Gen. 37:9-10). Here Joseph says "I had another dream and here the sun and the moon and eleven stars are bowing down to me". On which Jacob answers, "What is the dream you have dreamed? Are we to come, I and your mother and your brothers, and bow low to you to the ground?".

Rashi explains what Jacob had on his mind: "the mother (the moon) already died, while Jacob did not know that the moon refers to Bilhah (Rachel's maid) who raised Joseph as if she was his mother". As we stick together the words **אשר חלמת הבוא** (which you have dreamed.), they spell **רחל מתה** (Rachel died). Now we are looking for the word **בלהה** (Bilhah). The computer found two appearances of this word on the same page, both starting with the same letter **ב** next to the phrase **אשר חלמת** one is with the interval 99 and another with 156. We don't know exactly the meaning of 99 however 156 bears a direct reference to Joseph being the Gematria of his name (**יוסף**= 10+6+60+80 = 156).

There are hundreds of equally impressive examples which are not shown here due to the limited scope of this review. However, on the basis of the presented material we ask again the same question are the above systems a mere coincidence or they are deliberately planned? Now the skeptic concedes that the odds for each individual system are very small, however there are millions of different stories which one could look for so that occasionally some of them occur with small odds. Likewise in a lottery there are millions of players and few winners. The truth of the matter is that there are 34 people who have been searching mainly the book of Genesis by

computer for the last two years. They explored perhaps a few thousand words and systems while the success ratio was astounding. Nevertheless, to counter the above argument on a statistical basis one has to find "storyindependent" phenomena, i.e. something which could be checked automatically by computer and compared with other texts. The following example will be used to demonstrate such a general phenomena. This example is also important from a historical perspective since it marked the beginning of the "computer era" in the study of Torah. A "hidden" Aaron in Leviticus Our story starts somewhere in 1982. Abraham Oren from kibbutz Sde Eliahu was exploring manually whether the word **אהרן** (Aaron) is spelled out with equal intervals in the beginning of Leviticus.

Why Aaron and why in Leviticus? As everybody knows, Leviticus talks mainly about the work of the Cohanim the priests, and Aaron being the Cohen Gadol (the high priest) is the main hero of the book. Nonetheless, in the first open chapter (Parasha Ptucha) of Leviticus Aaron is not mentioned even once. Instead it repeats four times "the sons of Aaron". Abraham Oren was familiar with the work of Rabbi Weissmandel, so it was natural for him to suggest that Aaron is hidden inside the chapter in the way of equal intervals. And indeed he found quite a few. When he showed it to Dr. Eli Rips from Hebrew University, the latter typed this chapter on the computer and asked it to find all appearances of the four letter word **אהרן** in the chapter. The result of this search is shown in fig.6. There are altogether 25 hidden Aaron's not counting the explicit ones. The numbers which point to the circled **א**'s are the sizes of the intervals which should be counted from these **א**'s in order to obtain the word **אהרן**. The negative numbers correspond to the backward counting. In this example we are not selecting any specific interval like 26 or 50. Instead the computer checks all intervals from 2 to 235 (the maximal possible in this chapter), forwards and backwards from every letter **א** and tries to find the word **אהרן**. As Rips looked at the results he was overwhelmed by the large number of total appearances: 25. Indeed, the chapter is 716 letters long out of which there are 55 **א**'s, 91 **ה**'s, 55 **ר**'s and 47 **נ**'s. For a random distribution of these letters a statistical formula shows that the expected number of Aaron's in the text should be about 8 and that the probability of finding 25 or more Aaron's is about 1 in 400,000. That is, it would take 400,000 pages of text like the one in fig.6 until one would find 25 or more hidden Aaron's on a page.

A linguist could charge that the letters in the language are correlated so that the Hebrew of the Bible may "like" **אהרן** more then expected. Notice, however, that 12 Aaron's out of 25 are going backwards and it is not clear why the "forward" language should like them. And if it does, then equally well it should like other combinations of the four letters **א, ה, ר, נ**. So Rips took all 12 possible combinations (there are $2 \times 3 \times 4 = 24$, but forward and backward count as one) and performed with them the same experiment as with Aaron. In the lower part of fig.7 we see the results of the experiment. The word **אהנר** (meaningless) appears in the text 8 times and so does **ארהנ**. The other results 9,7,5 etc. center around 8 with a deviation of +3 in a complete agreement with the statistics and only the **אהרן** stands out.

The next experiment is shown in the upper part of fig.7. As well known, in Hebrew there is a short and full spelling. In Torah the same words sometimes are spelled full and other times short. If we change the spelling, the equal intervals become at once nonequal. Hence there is no reason

why the text should prefer **אהרן** in the form $\aleph(n) \eta(n) \rho(n)$ over $\aleph(n) \eta(n+x) \rho(n+y)$. Now we fix the numbers x and y and let the computer to search for Aaron with all possible n (i.e. from 2 to 235). The numbers x and y vary from 5 to 5 and for each pair x,y the total number of Aaron's is shown in the table. We see that these totals vary from 2 to 15 with the average 7.3 and the standard deviation 2.4. The number 25 corresponding to $x=y=0$ (i.e. equal intervals) is 7.4 standard deviations away from the average! So indeed, our text "likes" Aaron with equal intervals. But what about other words, maybe they exhibit the same phenomenon? And what about other texts? For comparison Rips took all 4 letter words, more precisely all 4 letter combinations in Hebrew alphabet. Since there are 22 letters, the total number of combinations is $22 \times 22 \times 22 \times 22 / 2 = 117,128$. Now you take any word out of 117,128, say **אבגד**, and do with it the same experiment as with **אהרן**. Namely, you let the computer find the number of times this word appears in our chapter and the expected number of appearances. Suppose that for **אהרן** these numbers are correspondingly 5 and 3. Then you compute the probability of having 5 or more appearances of the word instead of the expected 3. The result happens to be 0.185. Now turn to the upper table in fig.8. The vertical axis shows the number of appearances of a word while the horizontal the probability (on a logarithmic scale). The number 232 in the 6th row and 3rd column shows that 232 words out of 117,128 appeared 5 times in the text and the probability for them was around 1/10, and similarly for the other numbers. Thus the word **אבגד** was counted among the 232. As the probability decreases and the number of appearances increases, there are fewer and fewer words in the table. The position of the word **אהרן** is shown by the circle. Obviously Aaron is the winner of the competition! There is just one more word **יטעא** (meaningless) with the same probability 1/500,000 which appeared 6 times. Actually all words with probability less than 1/1,000 turned out to be meaningless. There are also 12 words which appeared more times than **אהרן**, but their probabilities are quite reasonable. Indeed, there are more or less frequent letters in our text. The words with very frequent letters should normally appear more times.

But what about other texts? Rips took for comparison a piece of the same length (716 letters) from the beginning of the novel "Hachnasat Kala" of the famous Israeli writer Shai Agnon (the only Israeli Nobel prize winner) and ran on it the same experiment with 117,128 words. We see that the distribution of numbers is the same as above with the only exception the circle which contained **אהרן** is now empty! And no meaningful word passed the limit of the probability 1/1,000. This proves once again that the whole phenomena of **אהרן** has nothing to do with the Hebrew language.

But maybe the comparison with Agnon is unfair since his is a different, "modern" Hebrew? Ideally, one should take a text which is first canonic, and second very close to Torah. It was Prof. BenChaim from the Academy of Hebrew Language who came up with a brilliant idea take the Samaritan Torah! Samaritans are thought to be the descendants of Kutim the nations brought into Israel after the exile of the 10 tribes (7th century B.C.). Although they had been influenced by Judaism, they did not become a part of the Jewish nation. There are still about 2,000 Samaritans living in Nablus. And they possess a Torah which differs from our tradition. Actually, there are numerous differences among their manuscripts, so it is hard to talk about an established version. Nonetheless, a few years ago two Samaritans brothers Tzdaka, published the most authentic

version of the Samaritan text and compared it with our Torah. What is shown in fig. 9 is the beginning of Leviticus. The right hand side is our Torah, the left is the Samaritan one. Our chapter with Aaron's consists of the first 14 verses. The differences between two texts are boldfaced. Besides an additional 20 letter phrase in the 10th verse there are 16 places where the texts disagree. But otherwise this is the same story and in translation it would read the same. So it was very interesting to see what effect these differences had on Aaron. And low and behold, they destroyed 22 out of 25 hidden Aarons! However also 7 new Aarons surfaced. Thus the total became 10 instead of 25 in complete agreement with the statistics since the expected number is about 8 with deviation of plus or minus 3.

At this point the skeptic is ready to admit that people could have done it deliberately. "You know", he says, "they had a lot of time to do this. The sages say that Rabbi Akiba used to count letters. So apparently there was such a tradition". Let us explore this line of thought. Suppose some people, say the priests themselves planted these Aaron's in the text. But for what purpose? To impress somebody later on? However, until discovered by Abraham Oren and Eli Rips this secret was absolutely unknown. Moreover, were it discovered 40 years ago nobody would be impressed by it. Indeed you should do all the comparisons to see how outstanding the phenomenon is and this was impossible before the advent of computers. Did the author(s) of the book anticipate the computer era? And then a technical question how did they do it? Suppose there was an existing text without Aaron's like the Samaritan Torah. Is it possible with a little editing to create the 25 Aarons? The author of this review actually tried to add another (26th) Aaron to the existing 25 with no avail. But even if this is possible, there is a limit of how many words one can hide in a meaningful text. The 25 4 letter Aarons put $25 \times 2 = 50$ constraints on the 716 letter text (i.e. the distance between א and ה is the same as between ה and ו and as between ו and י giving two constraints per word). It is hard to set a precise limit but we feel that one can't produce a meaningful story where 30% of its letters are tied up by constraints like those above. And this is not a question of personal ingenuity or whether the author had a computer at his disposal. The language has its set of words and grammatical rules, so mathematically speaking you are going to have more equations (constraints) than the unknowns (the words). Of course, if the author is creating the language simultaneously with the text then the above limit does not apply. These are indeed confusing questions.

So our skeptic backs up and suggests that maybe the whole system with Aaron's is just another coincidence. "After all, why did you take the first chapter and why Aaron? There are so many chapters and so many important words you could have chosen so that one success even with a ratio of 1/400,000 is not outstanding at all!". We could reply that Aaron is the most important word in Leviticus and intuitively the first chapter has preference over the other ones. However the whole story with Aaron's was brought here not for the sake of showing another oddity but rather to demonstrate some general phenomena.

The clustering effect

After the discovery of Aaron's, Rips obtained an electronic text of Genesis and started a systematic investigation. (It was only recently that the full electronic errorfree text of Torah became available to us). By the text of Torah, unless stated otherwise, we always mean the

traditional Ashkenazi Masoretic text as published in so called Koren edition. There is another text accepted among Yemenite Jews. These two versions were carried by two independent traditions for more than a thousand years. Yet, as we compare these texts, they differ only by 9 letters out of 304,805! Among the nine, there are 3 different letters in Genesis (of a total 78,064). Besides, there are several ancient manuscripts. One of them is called the Leningrad codex (because it is in the possession of a Leningrad library) and was copied 1,000 years ago in Egypt. As was shown recently by Dr. Mordechai Breuer in "Keter AramTzova" this text differs from the Koren edition by 130 letters. Almost all of these 130 letters are contradicted by the majority of other manuscripts and, most important, by the Masoretic instructions. Nonetheless the Leningrad codex is called the "scientific text" of Torah and is used by several universities for their databases. Clearly, even one missing or extra letter destroys the hidden words which "leap" over this letter. However the examples shown in this review appear in parts of Genesis which are away from the doubtful letters and hence are not affected by them.

So let us define the clustering effect. As we saw with Aaron's the word was spelled explicitly (4 times) in the chapter and at the same time it appeared there in a large concentration in the equal interval form. Rips wanted to check whether the same phenomenon occurs with other words. Since it was not feasible to scan all words, Rips started with the words in the beginning of Genesis. The text in fig.10 consists of Gen.1 and 2 as it appears in the Koren edition. It totals 2956 letters and has about 120 different words of the length greater than 2 (not counting different grammatical forms). Each word was run by the computer to find where it appears with equal intervals. The intervals n were taken in a range from 2 to some N , both positive and negative. The results of such a search for the word **עֵדֶן** (Eden) are shown in fig.10. The word **עֵדֶן** is spelled out explicitly in three places as shown by the rectangles. The circles show the hidden Edens and the numbers leading to **ע**'s indicate the appropriate intervals. In this case the range of intervals N was taken to be 120. The number N is chosen in such a way that there is a reasonable amount of hidden words. For example, if one choses $N=240$ there would be twice as many hidden Eden's mixing all over the text and it would be difficult to see the clustering. Likewise for $N=60$ it would be too few words to make statistical estimates. We see that there are 4 hidden Eden's on the first page and 4 on the second page. The story of the Garden of Eden is told in the verses Gen. 2:4-14 starting at the bottom of the second page. Here inside a segment of 379 letters 16 hidden Eden's appear!

What force has drawn them together? Maybe the 3 explicit Eden's increase the local density of the letters **ע**, **ד** and **ן** so that there are more chances for the hidden ones? A computation like the one performed for Aaron shows that the expected number of Eden's is about 5 and the probability of such a deviation is about 1 in 10,000. (We see another weaker cluster at the bottom of the third page where the Torah tells about the creation of the woman indeed she was intended to be the **עֵדֶן**=pleasure for the man).

In fig.11 we see a similar example with the word **הַנָּהָר** the river. The word is mentioned 4 times explicitly as shown by the rectangle frames. When run on the computer with intervals up to 80 it produces a cluster of 11 words over 2/3 of a page while on a usual page it appears about 3 times. Next, in fig.12 the word **מִקְוֵה** (gathering of water) is exhibited. There is a cluster of 10 words around the explicit **מִקְוֵה** while on the other pages the word appears once or twice. Note that this

time the hidden words do not cross the explicit one so that the letters of the explicit **מקוה** could not cause the cluster. Fig.13 demonstrates a similar effect with the word **מקום** surrounded by a cluster of 8 hidden words, while on the second and third page there are altogether 4 hidden words.

Especially interesting are the results for long words. Clearly, the longer the word is, the smaller are the chances to find it in a text with a given interval. In fig.14 three such words are shown **בהבראם** (as they were created), **החווילה** (Havilah) and **המוידם** (the dates). The six letter word **הבראם** was searched for by the computer over the whole book of Genesis (i.e. 78064 letters) with equal intervals in the range 300 to 300. It was found to appear 4 times one of them with the interval 176 clusters around the explicit word. Similarly the word **החווילה** in the same range appeared 6 times one of them with interval 167 clusters around the explicit word. The seven letter word **המוידם** was searched for in the book of Genesis with intervals from 10000 to 10000! It appeared only once, the interval being 70, and clusters right where the word is spelled explicitly. (By the way, there are exactly 70 days in a year called **מועדים** as defined in Lev. 23 52 Sabbaths, 7 days of Pesach, 1 day of Shavuot, 1 day of Rosh Hashana, 1 day of Yom Kippur, 7 days of Sukkot and one day of Shmini Atzeret). But what about other words? Obviously we cannot show here all of the results. However about 40% of the words in the above 3 pages produced a strong clustering effect, another 40% showed a moderate clustering and the rest no clustering.

Part of the clustering is effected by the noneven distribution of letters. For example, when the word **אדם** (Adam) is mentioned in Gen. 2:5,7 there is a nearby **אדמה** (the earth) which adds letters **מ,ד,א** to the text and increases the likelihood of the appearance of the hidden **אדם**. When for comparison we took a 3000 letter piece of text from the novel "Arie Baal Guf" (The bulky Arie) of Bialik, there was also a cluster effect although much weaker than in Genesis. Hence, in order to measure the "net" clustering Rips suggested comparing the equal intervals with the nonequal ones in the same text, as it was done with "Aaron" (see fig. 7).

The next question is how to measure the clustering quantitatively? The simplest way is to specify in advance a neighborhood of the explicit word and then check how many hidden words appeared in this neighborhood. It is clear, however, that for longer words the neighborhoods should be greater than for the shorter ones and hence it is preferable not to compare words of different lengths. Finally, a controlled experiment was run for all 3 letter nouns in Gen.1 and 2 altogether 50 words. The neighborhoods to be considered were 300 letters long (about 8 lines) and centered at the explicit words. The total number of hidden words in the neighborhoods was 370 versus the expected 300, which was 4 standard deviations away from the expectation. The results for nonequal intervals were about the average.

Next the same experiment was performed for the Samaritan version. Here the results for the equal and nonequal intervals were about the same as the expectation. Four standard deviations correspond to the probability of about 1/100000. This is indeed a very small number. However some statisticians may say that the text under investigation is too short. Besides, for 3 letter words the non equal interval test is very limited. That is, for the word **עדן** we consider the

sequences $\mathcal{Y}(n) \mathcal{T}(n+x) \mathcal{N}$ with fixed x and all possible n . The number x should be small so that the nonequal intervals would be a small perturbation of the equal ones. For example, if x varies between 5 and 5 we have only 10 different results to compare. If the word is longer, e.g. 5 letter word ABCDE, the perturbed sequences are $A(n)B(n+x)C(n+y)D(n+z)E$ so that with x,y,z , in the same range of 5 to 5 there is a sample of 1330 different results. Hence Rips suggested to check the clustering for 5 letter words over the whole book of Genesis. This requires a prohibitive amount of computations, so Rips restricted himself to all 4 letter nouns preceded by a definite article **ה** which are encountered in Genesis. The final list consisted of 86 words.

Next Rips has defined a probability function which measured the clustering for each word. The definition is too technical to be presented here. Roughly speaking, the function attains the values between 0 and 1, is uniform for a random text and becomes small when a hidden word with a short interval N appears close to the explicit one. Then for each word a "race" was performed in which the equal intervals competed with the nonequal perturbations. In the first "race" the numbers x,y,z were between 2 and 2 thus providing $5 \times 5 \times 5 = 125$ "runners". The probability function was measured and the "runners" with the smallest value would win.

The results of the 86 "races" were as follows. In 3 instances the equal intervals defeated the nonequal ones. The words were **המקנה** (the livestock), **החתמת** (the seal) and **הבהמה** (the domestic animal). For eleven more words the equal intervals were among the top 10% of the "runners". These results are not impressive at all since the probability that 14 out of 86 instances would be in the upper 10% is about 1/20. Next, the three winners were "allowed" to compete with about 5000 "runners". Namely, the range of x,y and z in the nonequal intervals was increased from [2,2] to [8,8] which produced $17 \times 17 \times 17 = 4,913$ "competitors". (It was too expensive to make such a "race" for all the words since it takes several hours of computer time to run a single word). The words **המקנה** and **החתמת** were champions also in the big race. Now the combined phenomena of 14 top 10% words and 2 top .02% ones has a probability of 1 over 30,000. The same experiment was performed also with the Samaritan text. Here only two words **הקדשה** (the harlot) and **המגדל** (the tower) were in the top 10% and no word entered the upper 1%. Thus the Samaritan text behaves like a "normal" one.

Our skeptic might be unimpressed by the probability of 1/30,000. Indeed, with Aaron's we already had 1/400,000. However this time the test was both word and segment independent. Namely, instead of a specific (though important) word Aaron we took a big "natural" sample and instead of the first chapter the whole book of Genesis. One also should bear in mind that the clustering is only one aspect of the infinite information hidden in Torah in the way of equal intervals. There is no clustering for "Torah" in fig.1 or for "Israel" in fig.2. King David is not mentioned explicitly in fig.3 so we lose another story and likewise for the "Temple" in fig.4 and "Bilhah" in fig.5.

One should really wonder that after all nontrivial patterns have been neglected there is still something to observe. In the next section we will demonstrate another general idea which is common to many words and patterns. The minimal intervals When the computer searches for a certain word with equal intervals in a wide range of numbers it will find the word many times. Some of the intervals may be of special interest like the numbers 50, 26 etc. But what shall we

do with the other ones? In the course of numerous experiments Rips observed that the short intervals tend to be more significant than the long ones, i.e. they appear more often in relevant places.

We will present here one example of this phenomenon. The text in fig. 15 consists of Gen.2 (this is an enlargement of the third page of fig.10). Verse 9 reads: "And from the ground Hashem God caused to grow every tree that was pleasing to the sight and good for food with the tree of life in the middle of the garden and the tree of knowledge of good and bad". The names of the trees however are not mentioned in the chapter. So Rips suggested that perhaps these names are hidden in equal intervals. The book of Yehuda Feliks "The fauna and flora in the Torah" lists the names of all the trees which are mentioned in Torah. And all of these names a total of 26, were found in the above chapter! Before the reader jumps out of his seat, let us explain that three or four letter words would normally appear with some intervals in a segment as long as ours (about 1000 letters). What is so exceptional here is that most of the intervals (except for ערמון and לבנה) are very short. There is no other segment in Genesis of such length which contains so many trees with intervals less than 20. Based on the density of the letters in the chapter one could estimate the probability of the "orchard" phenomenon the number is about 1 in 100,000!

Conclusion

We started with the "Torah" of Rabbi Weissmandel, went through the examples of "Israel", "King David", "Temple Torah", "Rachel with Bilhah", to "Aaron", then to the clustering effect in general and to the "orchard" and the minimal intervals phenomenon. There are many more fascinating examples and stories which could not be included in this limited review. A book with much of this material should soon be published in Israel. We hope that our skeptic also concedes that the equal interval phenomenon is not an imagination of a few "phony" people or a deliberate trickery with a computer but a reflection of a hidden design. We are far from understanding the rules of this design, in particular what stands behind the numerical values of all the different intervals?

In recent years there were some other coded systems discovered (or rediscovered) in the Torah. Let us mention the multiples of seven, where the key words in each chapter appear either 7 or 14 or 21 etc. times. Another rule discovered by the late Rabbi Suleiman Sasson states that for each word which is repeated in Torah more than 80 times, its 80th appearance is in a segment which talks about a promise, covenant, marriage or purchase (i.e. different types of contract). The distinction of the equal intervals is that they appear on the letter rather than the word level and that they contain apparently limitless information. But who made this design? Nachmanides writes in the introduction to his commentaries on Torah that Moses saw the Torah as a letter string of a black fire on the background of a white fire. This string of letters was not divided into words. As God dictated the Torah to Moses, he (Moses) wrote it accordingly in the form of words and chapters. As Maimonides states in the introduction to Mishne Torah, Moses wrote the Torah before his death one copy for each tribe and one to be kept in the Ark. It is believed that the modern Torah text is the exact copy of the original (modules maybe few letters, as suggested by the comparison of the Yemenite and Ashkenazi texts). This is what Judaism claims.

What do the Bible critics have to say? According to them Torah is a patchwork which consists of

pieces written in different times by different authors. These pieces allegedly were put together during or after the Babylonian exile and then canonized. For example they say that Gen.1 and Gen.2 were written by different authors because Gen.1 uses the name ALH'M=Gd while Gen. 2 the name Hashem Gd. Since there are hidden words like **בהבראם** in fig. 14 which connect Gen.1 and Gen.2 we should assume that they were built by the final editor. If one counts all the trees in fig.15, the most outstanding clusters like "Eden" and "the river" (fig. 10 and 11) and few other systems with probabilities less the 1/1000 the number of letters employed by the hidden words is about 30% of the total. Thus one has to believe that this editor with some small modification (and without any apparent reason) created all these codes? "It is possible", says our skeptic, "that the ancients possessed some secret knowledge which we cannot comprehend take for example the great pyramids or the temples of Inca". Whatever they knew, nobody would suggest that they could foresee the future (unless they had a time machine?).

We started with an example of Rabbi Weissmandel and we shall finish with another example of his. Everybody has heard the name Maimonides the greatest Jewish scholar and philosopher. In Hebrew his name is pronounced **רמבם**, the four letters being initials of Rabeinu Moshe Ben Maimon (Rabbi Moshe son of Maimon). Maimonides was born in Spain 851 years ago and later settled in Egypt where he became a court doctor of Tzalach EdDin. There he wrote his most important work the 14 books of Mishne Torah where he classified and clarified all of the 613 Commandments the 248 obligations and 365 prohibitions which are binding for every Jew.

Fig.16 shows the beginning of the Mishne Torah where Maimonides explains what is the origin of the Commandments and how they are divided among his 14 books. There is a remarkable parallel between Moses and Maimonides. They have the same name Moshe, Moses died at the same day he was born (Adar 7th) and so Maimonides died at the same he was born (Nissan 14th). They both lived in Egypt and performed marvels before the rulers of Egypt (Maimonides as a court doctor). Maimonides' Mishne Torah which is a full summary of Judaism parallels the Moses Mishne Torah, or Deuteronomy which is a summary of Torah (see again the beginning of the review for the explanation of the "Torah" with interval 49 in Deuteronomy). Furthermore, there is a popular saying **מי משה למשה לא כמשה** "from one Moses till another Moses there was nobody like Moses". Nachmanides (**רבמן**) who lived few decades after Maimonides claimed that he had found the latter once mentioned in the Torah. The verse Exod. 11:9 reads : "Now the Lord had said to Moses, Pharaoh will not heed you, in order that my marvels may be multiplied in the land of Egypt" (see fig. 17). In Hebrew the underlined phrase is **רבות מופתי בארץ מצריים**. The initials of these four words form the name **רמבם** (which by itself consists of the initials of the full name). "How beautiful", says our skeptic, "but you probably will find such **רמבם** on each page". We did check this is the single **רמבם** in the entire Torah spelled by the initials of the consecutive words! But this is only the beginning of the story. Forty years ago Rabbi Weissmandel came across this passage. And then he asked himself could it be that there is something else about Rambam hidden in a way of equal intervals? So he took the name of Rambam's greatest book Mishne Torah (spelled in Hebrew as **משנה תורה**) and started to search for it. Since he already had discovered the "Torah" system with intervals of 50 (corresponding to the 50 gates of wisdom) he was looking again for intervals of 50. And indeed, starting with **מ** of **משה** (Moses) in the above mentioned verse he found the word **משנה** with the interval 50. The

second part **תורה** appeared much lower again with interval 50. The large gap between **משנה** and **תורה** apparently puzzled him. He counted the number of letters between the **מ** of **משנה** and **ת** of **תורה** and it was 613 as the number of the Commandments. If one still wishes to know the probabilities the likelihood of such **תורה משנה** starting with a given **מ** is 1 in 186,000,000. You could of course try some other **מ**, say 10 possibilities for **מ** in a close neighborhood of the **רמבם**. And you could play with 613 counting them between the **ה** of **משנה** and **ת** of **תורה** or **מ** of **משנה** and **ה** of **תורה** and also include or exclude the first and the last letters in the counting, which gives you 6 possibilities. So with all this playing around you can increase the likelihood to 1 in 3 million. Now, what is the bottom line? Either the one who wrote the Torah knew 2,500 years in advance about Maimonides and Mishne Torah or the whole story is another coincidence with a probability of 1/3,000,000.

Unfortunately, when it comes to very small or large numbers people often lose common sense. Let us suggest a following mental experiment. One is offered the chance to play Russian Roulette in which he loads the cylinder of a pistol with one bullet out of 6 chambers, rotates the cylinder and shoots at his head. There is no other partner and one should repeat the game 81 times. If the person dies he dies. If he stays alive (and the chances are 1 in 3,000,000) he will have an exciting experience. Would our skeptic take the offer? Three thousand three hundred years ago there was another skeptic Pharaoh was his name. Our story in Ex. 1112 is told after Pharaoh had experienced nine plagues. He was still not convinced because, as Torah says, "The Lord had stiffened the heart of Pharaoh". Should one wait for the tenth plague?

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